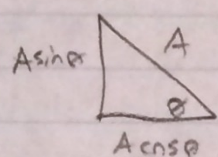
Physics Semester 1 Formulas

$r = \sqrt{x^2 + y^2}$ $\quad$ $x = r\cos\theta$

$\tan\theta = y/x$ $\quad$ $y = r\sin\theta$

$\vec{A} \cdot \vec{B} = AB\cos\theta$ $\quad$ θ = Angle in between

$\vec{A} \times \vec{B} = AB\sin\theta$



$$\vec{R} = \underbrace{\vec{R}_x}_{A_x + B_x} + \underbrace{\vec{R}_y}_{A_y + B_y}$$

$$\tan\theta = \frac{A_y}{A_x}$$

$$A = \sqrt{A_x^2 + A_y^2}$$

Dot products

$\hat{i} \cdot \hat{i} = 1$ $\quad$ $\hat{i} \cdot \hat{j} = 0$

$\hat{j} \cdot \hat{j} = 1$ $\quad$ $\hat{i} \cdot \hat{k} = 0$

$\hat{k} \cdot \hat{k} = 1$ $\quad$ $\hat{j} \cdot \hat{k} = 0$

Cross product

$\hat{i} \times \hat{i} = 0$ $\quad$ $\hat{i} \times \hat{j} = \hat{k}$ $\quad$ negative if reversed

$\hat{j} \times \hat{j} = 0$ $\quad$ $\hat{i} \times \hat{k} = \hat{j}$

$\hat{k} \times \hat{k} = 0$ $\quad$ $\hat{j} \times \hat{k} = \hat{i}$

$$\vec{V}_{Avg} = \frac{\Delta x}{\Delta t} = \frac{x_F - x_i}{t_F - t_i}$$

$$V_{instant} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

$$A_{average} = \frac{\Delta V_x}{\Delta t} = \frac{V_{xF} - V_{xi}}{\Delta t}$$

$$A_{inst} = \lim_{\Delta t \to 0} \frac{\Delta V_x}{\Delta t}$$

1. $V_F = V_i + at$
2. $x_F = x_i + V_i t + \frac{1}{2} a t^2$
3. $x_F = x_i + \frac{1}{2}(V_F + V_i) t$
4. $V_F^2 = V_i^2 + 2a(x - x_i)$

Free fall $a = -g$

in two directions? split into x and y components
- magnitude is $\sqrt{(V_{xF})^2 + (V_{yF})^2}$

Projectile motion

$a_x = 0$ $a_y = -g$

$V_{xi} = V_i \cos\theta$
$V_{yi} = V_i \sin\theta$

$V_{xF} = V_{xi} = V_i \cos\theta$
$V_{yF} = V_{yi} - gt$

$x_F = V_{xi} t$
$y_F = y_i + V_{yi} t - \frac{1}{2} g t^2$

Trajectory of projectile

$$y_F = x_F\left(\tan\theta - (x_F)\left(\frac{g}{2v_i^2\cos^2\theta}\right)\right)$$

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

$$h_{max} = \frac{v_i^2 \sin^2\theta}{2g}$$

Circular motion

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

$$T = \frac{2\pi r}{v} \quad (rev/sec)$$

$$V = \sqrt{gr} \quad \text{since} \quad A = \frac{v^2}{r} = g$$

$$a_{total} = a_{rad} + a_{tan}$$

$$a_{rad} = \frac{v^2}{r} \qquad \theta = \tan^{-1}\left(\frac{a_{tan}}{a_{rad}}\right)$$

$$a_{tan} = \frac{d|\vec{v}|}{dt}$$

Force

$$F = ma \rightarrow F_x = ma_x, \quad F_y = ma_y$$

$$a_x = \frac{\Sigma F_x}{m} \qquad a_y = \frac{\Sigma F_y}{m}$$

$$a_{total} = \sqrt{(a_x)^2 + (a_y)^2}$$

$$a_{direction} = \tan^{-1}\left(\frac{a_y}{a_x}\right)$$

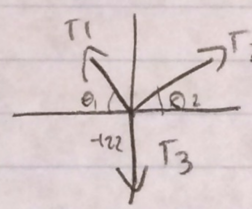
$\star$ Weight $= f_g = (m)(g)$ — when given weight, g is included — don't use $\boxed{g}$

3rd law

$$\vec{F}_{1m2} = -\vec{F}_{2m1}$$

Tensions → force on ropes
— all forces add to 0 when balanced

ex:



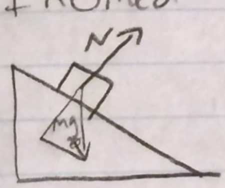
	X	Y
T_1	$-T_1\cos\theta_1$	$T_1\sin\theta_1$
T_2	$T_2\cos\theta_2$	$T_2\sin\theta_2$
T_3	0	-122 given

$$\Sigma F_x = -T_1\cos\theta_1 + T_2\cos\theta_2 = 0$$
$$\Sigma F_y = T_1\sin\theta_1 + T_2\sin\theta_2 - 122 = 0$$

— solve ΣF_x in terms of T_2 (or others)
— sub into ΣF_y

$$T_1\sin\theta_1 + \left(\frac{T_1\cos\theta_1}{\cos\theta_2}\right) = 122$$

Inclined plane



— Normal force perp to surface
— mg straight down — split into $mg\sin\theta$ for y and $mg\cos\theta$ for x
— Normal force is equal to the sum of downward forces in y direction
— if going down → has force in x direction equal to Ma_x
— no y direction force (cancel)

$$\Sigma F_x = mg\sin\theta = Ma_x$$
$$\Sigma F_y = N - mg\cos\theta = 0$$

$\star$ $\star$ — solve for $\boxed{a_x}$ this way
— get N force by solving for mg in ΣF_x and sub into ΣF_y $\quad a_x = g\sin\theta$

Object sliding down inclined plane formulas

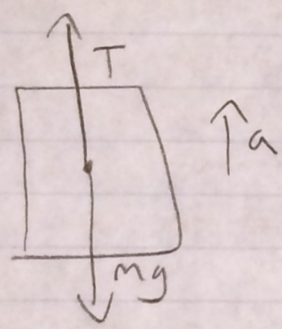
$d = \frac{1}{2} a_x t^2$ (use to solve for t after a_x found in $\Sigma F_x = ma_x$)

d = hypotenuse of inclined plane

$t = \sqrt{\frac{2d}{a_x}} = \sqrt{\frac{2d}{g\sin\theta}}$ because $a_x = g\sin\theta$

$V_{xF}^2 = 2a_x d \longrightarrow V_{xF} = \sqrt{2gd\sin\theta}$

Ex



$\Sigma F_y = T - mg = ma_y$

$T = ma_y + mg$

$T = mg\left(\frac{a_y}{g} + 1\right)$

$T = F_g\left(\frac{a_y}{g} + 1\right)$

- if a_y was downward, it would be negative

Chapter 5 (Friction)

$F_s = \mu_s n$

$F_k = \mu_k n$

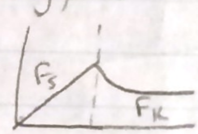
μ = coefficient of friction

s = static (not moving)

k = kinetic (moving)

$k < s$

- friction acts in direction <u>opposite</u> to force (along x axis)



$\mu_k = \frac{V_{xi}^2}{2g x_F}$

- you can obtain μ_s by measuring the angle of incline (or force) at which slipping just occurs

$$f_s = n\tan\theta \quad \rightarrow \quad \mu_s \cancel{n} = \cancel{n}\tan\theta$$
$$\mu_s = \tan\theta$$

Max velocity if Tmax is Tmax

$$V_{max} = \sqrt{\frac{T_{max}\cdot r}{m}}$$

When f_s turns into $f_k = f_{s\,max}$

$$F_{s\,max} = \mu_s n = m\cdot\frac{v^2_{max}}{r}$$

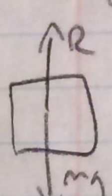
Banked road equation:

$$\tan\theta = \frac{v^2}{rg}$$

Resistant forces

- at low speeds the resistive force is proportional to the velocity (related by constant B)

$$\vec{R} = -b\vec{v}$$



$$\Sigma F_y = mg - bv = m\frac{dv}{dt}$$

$$V = \frac{mg}{b}\left(1 - e^{-bt/m}\right) = V_t\left(1 - e^{-t/\tau}\right)$$

Model 2

→ Resistive force is proportional to object speed squared

$$R = \frac{1}{2} D \rho A v^2$$

D = drag coefficient
ρ = density
A = crosssectional area of object
V = speed

So

$$V_t = \sqrt{\frac{2mg}{D\rho A}}$$

Work

$W = F \cdot d = Fd\cos\theta$

$W_g = mgr\cos\theta$

Spring work

$F_s = -kx$ $\qquad$ k = spring constant

$W_s = \frac{1}{2}kx^2$

$\Delta K = \text{Work} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

Potential Energy

$U_g = mgy$

elastic $U_g = U_s = \frac{1}{2}kx^2$

$pe_i + ke_i = pe_f + ke_f + F_k d$

power

$P = \frac{dE}{dt} = \frac{W}{dt}$

Momentum

$\vec{p} = m\vec{v}$

$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{1f} + \vec{p}_{2f}$

$I = \Delta p$

1D collisions

Inelastic

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_F$$

so

$$v_F = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

Elastic (both p and ke conserved)

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1F} + m_2 v_{2F}$$

and

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1F}^2 + \frac{1}{2} m_2 v_{2F}^2$$

$$\Big\downarrow \text{rework}$$

speed before = − speed after

$$v_{1i} - v_{2i} = -(v_{1F} - v_{2F})$$

$$v_{1F} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2 m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2F} = \left(\frac{2 m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

2D collisions

- separate into x and y

$$m_1 v_{1xi} + m_2 v_{2xi} = m_1 v_{1xF} + m_2 v_{2xF}$$

$$m_1 v_{1yi} + m_2 v_{2yi} = m_1 v_{1yF} + m_2 v_{2yF}$$

elastic 2D = $\frac{1}{2}m_1 v_{1i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$

—only applies to elastic (KE conserved)

Center of Mass

$$X_{cm} = \frac{m_1 x_i + m_2 x_i}{m_1 + m_2}$$

x = center of mass of each part

$$\vec{r}_{cm} = \frac{\sum_i m_i \vec{r}_i}{M}$$

$$X_{cm} = \frac{1}{M}\int x\,dm$$

$$y_{cm} = \frac{1}{M}\int y\,dm$$

then

$$\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\,dm$$

Motion

velocity of CM

$$\vec{V}_{cm} = \frac{1}{M}\Big(\underbrace{\sum_i m_i \vec{v}_i}_{=\sum \vec{p}_i}\Big)$$

$$\vec{V}_{cm} = \left(\frac{1}{M}\right)\left(\vec{P}_{total}\right)$$

Acceleration of CM

$$\vec{a}_{cm} = \frac{1}{M}\Big(\underbrace{\sum_i m_i \vec{a}_i}_{=\sum \vec{F}_i}\Big)$$

$$\vec{a}_{cm} = \left(\frac{1}{M}\right)\left(\sum_i \vec{F}_i\right)$$

Net force in system due only to external forces

$$\sum \vec{F}_{ext} = (M)(\vec{a}_{cm}) = \frac{d\vec{P}_{total}}{dt} = \frac{\Delta \vec{P}_{total}}{\Delta t}$$

$$\Delta \vec{p}_{total} = \vec{I} \text{ (impulse)}$$

Chapter 10 (Rotational Motion) (rigid objects)

$s = r\theta$ s = arc length, r = radius (m) θ in radians

$\theta \text{(rad)} = \left(\frac{\pi}{180}{}^\circ\right)$ 1 rotation = 2π radians

Angular speed (ω)

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t} \qquad \omega_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Angular acceleration (α)

$$\alpha_{avg} = \frac{\Delta\omega}{\Delta t} \qquad \alpha_{inst} = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

Constant angular acceleration

1. $\omega_F = \omega_i + \alpha t$
2. $\theta_F = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$
3. $\theta_F = \theta_i + \frac{1}{2}(\omega_i + \omega_F)t$
4. $\omega_F^2 = \omega_i^2 + 2\alpha(\theta_F - \theta_i)$

tangental velocity of a point (v)

$v = r\omega$ so $\omega = \frac{v}{r}$

accelerations

$a_t = r\alpha$

$a = \sqrt{a_t^2 + a_c^2}$

$a_c = r\omega^2$ or $r\left(\frac{v}{r}\right)^2 = r\left(\frac{v^2}{r^2}\right) = \boxed{\frac{v^2}{r}}$

Rotational KE

$K_R = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2$

$I = \sum_i m_i r_i^2 = \int r^2 dm$

$K_R = \frac{1}{2} I \omega^2$

$KE_{total} = KE_{rotational} + KE_{translational\ at\ center\ of\ mass}$

$K_{tot} = \frac{1}{2} I\omega^2 + \frac{1}{2} m v_{cm}^2$

Values for I based on shape

Thin cylindrical shell $I_{cm} = MR^2$

Hollow cylinder $I_{cm} = \frac{1}{2}M\left(R_1^2 + R_2^2\right)$

Solid cylinder (disc) $I_{cm} = \frac{1}{2}MR^2$

Rectangular plate $I_{cm} = \frac{1}{12}M\left(a^2 + b^2\right)$ $a = L$ $b = w$

rod w/ rotation in center $I_{cm} = \frac{1}{12} ML^2$

rod w/ rotation at end $I_{cm} = \frac{1}{3} ML^2$

Solid sphere $I_{cm} = \frac{2}{5} MR^2$

hollow sphere $I_{cm} = \frac{2}{3} MR^2$

Torque

T = tendency of a force to rotate an object

d = moment arm

$T = rF\sin\theta \longrightarrow T = Fd$

– magnitude → direction is right hand rule

$T_{net} = T_1 + T_2 = F_1 d_1 - F_2 d_2$

$\vec{T} = \vec{r} \times \vec{F}$

Newton's second law

$\Sigma T_{ext} = I \cdot \alpha \longrightarrow \Sigma \vec{T}_{ext} = I \cdot \vec{\alpha}$

$\Sigma \vec{F}_{ext} = 0$ and $\Sigma \vec{T}_{ext} = 0$

Energy

$W = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2 = \Delta K_R$

Power $= T \cdot \omega$

$dW = \vec{F} \cdot d\vec{s} = F(r\sin\theta)\, d\theta$

Non isolated system

$\vec{L}$ = instantaneous angular momentum

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\Sigma \tau = L_{total}$$

$$L = mvr|\sin\theta|$$

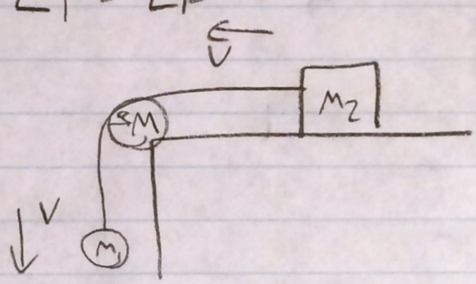
$$\Sigma \tau = I\alpha$$

$$\Sigma \vec{\tau}_{ext} = \frac{d\vec{L}_{total}}{dt}$$

$$\vec{L} = \left(\sum_i m_i r^2\right)\omega \longrightarrow \vec{L} = I\omega$$

Isolated $\rightarrow$ $\vec{L}_i = \vec{L}_F$

Pulley system



$$L_{tot} = m_1 VR + m_2 VR + MvR = (m_1 + m_2 + M)VR$$

$\Sigma \tau_{ext}$ = sum L_{tot}

$\downarrow$ derivative

$$mgr = (m_1 + m_2 + m)R(a)$$

$$a = \frac{m_1 g}{m_1 + m_2 + M}$$

Conservation of angular momentum

$$\Sigma \tau_{ext} = \frac{d\vec{L}_{tot}}{dt} = 0 \qquad I_i \omega_i = I_F \omega_F$$

end of the mass

(ma)

Chapter 11

$$\vec{F}_{12} = -G\left(\frac{m_1 m_2}{r^2}\right)\hat{r}_{12}$$

$G = 6.673 \times 10^{-11}$

$$U_g = \frac{-G m_1 m_2}{r}$$

Kepler's Laws

1st -elliptical orbits

$$e = \frac{c}{a}$$

$a(1+e)$ = further distance

$a(1-e)$ = closer distance

2nd the radius vector drawn from the sun to any planet

$L = \vec{r} \times p$ = $mvr\sin\theta$

3rd Square of the orbital period of any planet is proportional to the cube of the semimost axis of the elliptical orbit

$T^2 = K_s a^3$ $K_s = 2.97 \times 10^{-19}\ s^2/m^3$

Interplanetary travel

$$a = \frac{(R + R')}{2}$$

Time to fly

$$T = \sqrt{\frac{k_s a^3}{2}}$$

$K_s = 1$ when using AU units

Speed

$$G\frac{M_{planet}\, m}{r^2} = m\left(\frac{v^2}{r}\right)$$

solving for v

$$v = \sqrt{\frac{(G)(M_{planet})}{r}}$$

$$V_{escape} = \sqrt{\frac{2g(M_{planet})}{R_{planet}}} = \sqrt{2gR_{p}}$$

$$V_{orbit} = \sqrt{(g)(r_{planet})}$$

Critical radius at which escape speed equals c is called the schwarzschild radius R_s

$$R_s = \frac{2GM_s}{c^2}$$

$$T = 2\pi\sqrt{\frac{(R)^3}{GMe}}$$

$F(s) = -kx$ Restoring Force

$a_x = -\frac{k}{m}x$ simple harmonic motion

$x(t) = A\cos(\omega t + \phi)$

A = amplitude = max x or $-x$

ω = angular frequency = $\omega = \sqrt{\frac{k}{m}}$

ϕ = phase constant = shifted from 0

$T = \frac{2\pi}{\omega}$ or $2\pi\sqrt{\frac{m}{k}}$

$F = \frac{1}{T}$ or $\frac{\omega}{2\pi}$ or $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ unit = Hz

$x(t) = A\cos(\omega t + \phi)$

$v(t) = -\omega A\sin(\omega t + \phi)$

$a(t) = -\omega^2 A\cos(\omega t + \phi)$

$v_{max} = \omega A = \left(\sqrt{\frac{k}{m}}\right)(A)$

$a_{max} = \omega^2 A = \left(\frac{k}{m}\right)(A)$

Energy

$E_{total} = \frac{1}{2} k A^2$

$v = \pm \omega \sqrt{A^2 - x^2}$ or $\pm\left(\sqrt{\frac{k}{m}}\right)\left(\sqrt{A^2 - x^2}\right)$

Simple Pendulum

Small angles $\sin\theta = \theta$

$\omega = \sqrt{\frac{g}{L}}$

$T = \frac{2\pi}{\omega}$ or $2\pi\sqrt{\frac{L}{g}}$

Physical pendulum
- axis not through CM

$T = 2\pi\sqrt{\frac{I}{mgd}}$

Damped oscillations

$R = -bv$

$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$ → A

$x = Ae^{-(b/2m)t} \cos(\omega t + \phi)$

$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$

Forced oscillations

$$F_0 \sin \omega t - b\frac{dx}{dt} - kx = m\frac{d^2x}{dt^2}$$

$$x = A\cos(\omega t + \phi)$$

$$A = \frac{F_0/m}{\sqrt{(\omega^2-\omega_0^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$$

Energy

$$Ke = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$

$$Pe = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$$

$$E = U + K = \frac{1}{2}kA^2$$

use to find $v = \pm\omega\sqrt{A^2 - x^2}$

Chapter 13

Function of wave

$y(x,t) = y(x \pm vt)$ — $-$ is to right $+$ is to left

$$V = \sqrt{\frac{T}{\mu}}$$

T = tension

μ = linear mass density

Wave number = $k = \frac{2\pi}{\lambda}$

angular frequency = $\omega = \frac{2\pi}{T} = 2\pi f$

speed = $v = \lambda f$

$T = \frac{1}{F}$

new wave function

$$y = A \sin(kx - \omega t + \phi)$$

$K_e = \frac{1}{2}\mu\omega^2 A^2 \cos^2(kx - \omega t)$

$U_g = \frac{1}{4}\mu\omega^2 A^2 \lambda$

$E_{tot} = U + K = \frac{1}{2}\mu\omega^2 A^2 \lambda$

Power = $\frac{1}{2}\mu\omega^2 A^2 v$

Speed of sound

$$V = 331\sqrt{1 + \frac{T_c}{273}}$$

Doppler effect

$$f' = f\left(\frac{v + v_o}{v - v_s}\right)$$

V_o = speed of detector
V_s = speed of source
V = speed of wave

- if coming closer → increases f

Chapter 14

$y_1 + y_2 = y = 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)$

if $\phi = 0$ then $\cos\left(\frac{\phi}{2}\right) = 1$

when $\phi = \pi \rightarrow$ destructive interference or odd mult of π

when $\phi = 0$ or even multiple of $\pi \rightarrow$ constructive

Opposite direction travel

$A\sin(kx \pm \omega t)$

$y = (2A\sin kx)(\cos \omega t)$

$\cos \omega t = 1$ at max

Nodes $= 0 \ldots\ldots \frac{n\lambda}{2}$

antinode $= \frac{\lambda}{4} \ldots\ldots \frac{n\lambda}{4}$